# U.S. DEPARTMENT OF COMMERCE NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION NATIONAL WEATHER SERVICE NATIONAL METEOROLOGICAL CENTER

**OFFICE NOTE 252** 

Wave Number Verification of NMC's 7L PE Model

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This is an unreviewed manuscript, primarily intended for informal exchange of information among NMC staff members.

#### 1. INTRODUCTION

This study expands the National Meteorological Center's verification program for primitive equation models into the domain of wave numbers. The specific model used for this study is explained in more detail in Section 3, but similar techniques will be used for future operational models. Selected fields from the forecasts and their verifying analyses are decomposed by harmonic analysis into wave components. These sets of components are then compared to determine how well the model predicted each wave. Section 3 explains this technique in detail.

Fundamentally, this project was exploratory; the objective was to reveal and document the model's normal behavior, while probing the etiology of that behavior only in passing. Occasional discussions of causal relationships are therefore hypothetical-promising avenues for future inquiry. It is expected that the statistical approach described in this paper will be useful in evaluating modifications to the model, changes in data sources, or adjustments to the initialization technique.

In addition to the presently available statistics, wave number verification gives new information on the model. In particular, several of the model's characteristics are readily apparent: 1) the model has a general tendency to underpredict wave amplitude; 2) the model predicts waves, even the long waves (1-3) westward of their true position; 3) predicted mean, or zonal, flow is generally too weak. Each of these points is discussed at length in the body of this paper.

#### 2. BACKGROUND

The need to evaluate the performance of numerical models in the domain of wave numbers has been recognized for some time. Until now the NMC verification techniques used for numerical models have been continuations and refinements of methods previously used for manually-produced forecasts. These basically consist of S1 scores (Tweles and Wobus, 1954), RMS error calculations, and mean error maps, for various variables and areas. NMC's verification and evaluation programs were documented by van Haaren (1978), and are included in NMC's Annual NWP Progress Reports (Marks, 1979). The techniques involved provide a means for monitoring the performance of a model and analyzing the effect of changes thereto. Similar techniques are used by other centers, as pointed out by Dobryshman (1972) in his review of most of the major meteorological centers. As useful as these verification systems have been, they nevertheless fail to provide quantitative measures of the scale-dependence of model errors.

Evaluating forecasts in terms of wave structure is not a new idea. To date, however, the technique has not been widely used to define the characteristics of an operational model. Leith (1974) used spherical harmonics for evaluation purposes, but the model used was an experimental barotropic. Using another technique, Houghton and Irvine (1976), and Baumhefner and Downey (1978) made wave comparisons between models on a case study basis. Various researchers at NMC have also used wave comparisons in several case study projects. While these are certainly interesting and may even lead to the discovery of problems in the model's physics, the comparisons cannot define the model's normal or "average" behavior. Small, but consistent, errors simply cannot be identified by these methods. Pratt (1979) did investigate the normal behavior of general circulation models (from NCAR and

GFDL) but he was concerned with the long-range behavior. While the method used here is similar to Pratt's, I will be dealing with a model designed to produce short-range forecasts, not climatological simulations.

#### PROCEDURES

The following study was based on forecasts from the operational PE model described by Shuman and Hovermale (1968), and updated by Stackpole (1978). The analysis used for comparison with the model forecasts was the operational "Hough" analysis. This is the global analysis system described by Flattery (1971). The Hough analysis is taken as "truth" and any conclusions are made relative to the harmonic analysis of these fields.

Because the operational model is based on a polar stereographic projection, 381 km between grid points at 60°N, some interpolation is necessary. A simple bi-linear interpolation is used to obtain values on constant latitude circles, every 2.5° from 0°N to 87.5°N. On each latitude, points are spaced every 5° of longitude. A zonal harmonic analysis is them performed. Because most of the variance is contained in the first few waves, only wave numbers 1-15 are retained for analysis. The identical procedure is then applied to the operational analysis, which has been interpolated to the same grid.

As a result of the harmonic analysis, the field is expressed as:

$$X_i = \overline{X} + \sum_{i=1}^{P/2} (A_i \sin(\frac{2\pi}{P} it) + B_i \cos(\frac{2\pi}{P} it))$$

where  $\overline{X}$  is the mean value for the chosen latitude, i the wave number, P the number of points around a latitude circle (72), and t the point (between 1 and P) where the value is being determined. After the coefficients (Ai's and Bi's) have been calculated, it is possible to analyze characteristics of individual waves, i.e., each wave may be expressed as;

where  $Ci = \sqrt{Ai^2 + Bi^2}$  is the amplitude, and

the phase angle. To analyze the model behavior over a long period, there are several choices. First, on a daily basis, it is possible to compare the amplitudes for corresponding waves, i.e., get (Ci<sup>o</sup>-Ci<sup>F</sup>), the amplitude error (where Ci<sup>o</sup> is the observed amplitude, Ci<sup>F</sup> is the forecast amplitude). It should be noted that this amplitude (Ci<sup>o</sup>-Ci<sup>F</sup>) is not the amplitude of the ith wave in the error field because the phase angles differ. To describe the error field, the components must be subtracted, and the amplitude calculated, i.e., the amplitude of wave number i of the error field is:

$$\sqrt{(\overline{A_i^o} - \overline{A_i^F})^2 + (\overline{B_i^o} - \overline{B_i^F})^2}$$

Over long periods both  $(\overline{\text{Ci}^{\circ}-\text{Ci}^{\text{F}}})$  and  $\sqrt{(\overline{\text{Ai}^{\circ}-\text{Ai}^{\text{F}}})^2 + (\overline{\text{Bi}^{\circ}-\text{Bi}^{\text{F}}})^2}$  will approach zero, unless there is a bias in the model.

Analyzing phase angle differences is more difficult. The phase angles calculated daily are for phase relative to 0° longitude. As such they range over a full 360°. They cannot, therefore, simply be averaged, for the average of 1° and 350° turns out to be 180° which is misleading. It is possible, of course, to calculate the error in phase angle from the mean observed and forecast components, i.e.,

$$\theta^{\circ} = \frac{P}{2\pi} \operatorname{arctan}(\overline{A_i^{\circ}}/\overline{B_i^{\circ}})$$
 $\theta^{\circ} = \frac{P}{2\pi} \operatorname{arctan}(\overline{A_i^{\circ}}/\overline{B_i^{\circ}})$ 

Obtaining a daily phase angle error  $({\bf \Theta}^F - {\bf \Theta}^O)$  and expressing it in the range -180° to +180° provides the most useful value. On a daily basis, the error should be small but the monthly average will be meaningful.

It is important to note here that long-term patterns are being studied, that is, individual days and even weeks are of limited interest. This study is not concerned with the model's behavior in unusual cases, but with systematic behavior linked intrinsically to the model's configuration. Should other models need to be evaluated, similar techniques may be used and the results compared with those reported here.

## 4. YEARLY AVERAGE AMPLITUDE

Yearly averages for 1979 were obtained from the NMC archives. Data fields were available only from the 36-hour forecasts and only at five levels (1000 mb, 850 mb, 500 mb, 250 mb, and 100 mb). These data were decomposed for both synoptic times (0000 GMT and 1200 GMT); 637 distinct forecasts were averaged. This type of analysis is useful in determining any bias in the model, i.e., comparing the forecast amplitude (for each wave) with the observed amplitude (for that wave). It cannot show any seasonal variations.

The bias in the wave amplitudes is calculated by averaging the daily amplitudes of the observed and forecast waves. Only certain levels and latitudes will be presented here. Graphical results are presented at latitudes 30°N and 45°N in the mid-latitudes, and 60°N for the high latitudes. The levels presented are 1000 mb and 500 mb. The 6 graphs in Figure 1 show the observed and forecast amplitude patterns are quite similar at all latitudes. For all of these latitudes the forecast amplitude is less than the observed. This type of behavior was noted by Pratt (1979). Tables 1-5 give the observed and forecast amplitudes for all levels and a range of latitudes.

Looking at the various wave numbers, certain characteristics show up. The longest waves, in particular, are not especially well forecast. The 60°N 500 mb field (Fig. 1F) as well as all the fields at 30°N have some of their largest errors in wave number one. A very peculiar feature is the behavior of waves two and three. Wave two seems to have a very large, consistent error, while wave three has an error smaller than either two or four. This is shown best for 45°N, 500 mb, (Fig. 1E), but other graphs show the same tendency. Somerville (1980) recently concluded that errors in the tropics quickly propagate northward and become concentrated in the ultra long (1-3) waves. This was also noted in the earlier work by Baumhefner and Downey (1978), dealing with the 1972-73 version of the PE. The operational model, since it was extended to the equator in 1974, should contain enough tropical information, however, to prevent significant errors at these latitudes until the 3-4 day forecast.

Surprisingly, the 1000 mb charts all show the model to be quite good in forecasting amplitudes of shorter wavelengths. In fact, the errors are less than at higher levels. This is probably due to the relatively dense observation network that allows for resolution of these waves, that is, with more data, the analysis can put more energy into short waves. It should be remembered that these statistics come from a fairly short (36-hour) forecast, so the initial conditions will still be prominent in the forecast field. It cannot be inferred from Fig. 1A-1C that the entire forecast is actually better at the surface than at 500 mb. In fact, the S1 scores (Marks, 1979) indicate the opposite. This only indicates that the waves are resolved well enough at the surface so that the model, with its boundary layer physics and orography, can maintain the amplitude of these waves; there is less wave-related bias in the model at the 1000 mb level. The absolute error remains relatively

constant from wave 7 to wave 15. However, the observed amplitude is constantly decreasing in this range, so an ever larger proportion of the wave is being lost.

Looking at the graphs for January and July (Figs. 2A, 2B), seasonal differences may be inferred. At 45°N, (Fig. 2B), one aspect that stands out is the much higher amplitude of the waves in January versus July. It is also clear that the forecasts are better in January than in July. This is verified by statistics published by NMC (Marks, 1979) showing that the S1 scores are better in the winter than in the summer. In fact, most of the yearly bias in waves 3-9 can be attributed solely to the errors in the summer months. The data from 10°N (Fig. 2A), however, show a consistent pattern of errors for larger wave numbers. Surprisingly, here in the tropics, the very long waves have slightly greater amplitude in the summer than in the winter. It also appears that the model overforecasts these waves in the winter but underforecasts them in the summer.

Certain general observations may be made about all these graphs: 1) the average bias for the very long waves is not better, in an absolute sense, than for the shorter waves, 2) high amplitude (winter) waves are forecast relatively better than low amplitude (summer) waves, and 3) the model's bias is toward underforecasting rather than overforecasting.

### 5. YEARLY AVERAGE PHASE ANGLE ERRORS

Daily errors in phase angle are calculated by subtracting the observed phase angles from the forecast phase angles. These are then averaged over the entire year (Fig. 3).

To avoid problems due to the cyclic nature of phase angles, it is assumed that for any one day the phase angle error only varies between  $+90^{\circ}$  to  $-90^{\circ}$ . It must be remembered that these are phase-angle errors for that wavelength,

and are not expressed as errors in degrees of longitude. Angles in degrees of longitude could be obtained by dividing these values by the wave number. For example, wave number 6 will always require  $(360^{\circ}/6)=60^{\circ}$  of longitude for a complete cycle. Thus a 180 phase angle error means that the wave is actually only  $(180^{\circ}/6)=30^{\circ}$  of longitude out of phase. These average errors are shown for all latitudes and all levels in Fig. 3.

The most striking feature is the virtual absence of positive average errors. That means that the forecast normally moves medium and short waves too slowly at all latitudes. The longest waves, however, may be retrograding too quickly at higher latitudes. This is in contrast to Pratt's (1979) findings that the planetary waves were not forecast sufficiently westward in the GFDL and NCAR models. These observations are supported to some degree by Baumhefner and Downey (1978) in their six case study. Although there was considerable variability in the way that the planetary waves were handled, the models did forecast long waves too far to the west more often than forecasting them too far east. For the NMC model, this seems to be a definite bias with even the longest waves tending to be forecast farther west than the observed wave. There are small areas where the forecast was too fast, but these are rare and appear to be random.

Conversely, the negative errors show some definite characteristics. The most noticeable are the large values in the tropics and high latitudes. For wave numbers over twelve, phase angle errors of  $20^{\circ}-40^{\circ}$  are common throughout the domain. This observation indicates clearly that the model is designed for the mid latitudes and for synoptic scale waves. The mid latitudes do not display large values until wave number 10 and above. The very long waves (wave numbers 1-3) do not show significantly smaller errors (in an absolute sense) than waves 4-9, despite the fact that they should be represented

better in the analysis. Also, it is worthwhile remembering that a one-degree error in these waves is a much longer distance than a one-degree error in shorter wavelengths. Thus, equal phase angle errors actually mean that the forecast for the longer wave was worse than the forecast for the shorter wave. This tendency for the model to forecast eastward-moving planetary waves too slowly and with too small an amplitude was also noted by Baumhefner and Downey (1978) in their six case study.

#### 6. ERROR IN ZONAL MEAN

Simply looking at the errors in the waves does not tell the whole story. It is also worthwhile to examine the errors in the zonal mean (wave number 0) (Fig. 4 and Table 10). In addition to giving insight into the mean flow, this graph provides a cross section of the equator-to-pole wave structure. The normal slope of the pressure surfaces may be regarded as an equator-to-pole meridional wave. The fact that the tropical heights are too low and the polar heights too high has the effect of reducing the amplitude of this meridional wave (especially at 250 mb and above). But the large area of positive errors in Fig. 11 means that the mean value is increased. Because this increase accompanies a reduction in wave amplitudes (Fig. 1) it is possible to infer that the model is erroneously converting zonal mean kinetic energy (wave amplitudes) into zonal mean potential energy (a higher mean value). From the chart it is still clear that the model does the best in the mid-latitudes.

Perhaps the most likely mechanism causing this pattern is in the parameterization of diabatic heating. Indeed, the thickness pattern of the error in the zonal component indicates that the tropics are not receiving enough heating while the poles are receiving too much. So, increasing both the short-wave heating (which dominates at the tropics) and the long-wave

cooling (which dominates at the pole) should help. Actually, during the months of May-June 1979 (Fig. 5A), a cooling rate of 1.44°C/day was used, while 0.8°C/day was used for the other months (the value of the cooling term was reduced to correct a general decline of potential temperature in longer-range forecasts). This had the expected result of lowering the heights by about 10 meters. Thus, the average forecast error in the tropics was larger while the forecast for the polar region improved. While the cooling term affects the entire atmosphere equally, the short-wave heating tends to affect the tropics much more than the pole. Thus, increasing this heating could help eliminate the problem in the tropics without degrading the polar regions.

It should also be pointed out that changes in the heating parameterizations will not be manifested solely in the thickness fields. Heating and circulation fields are intertwined and changes in one will lead to changes in the other. It would not be surprising, therefore, for additional thermal forcing to lead to higher wave amplitudes on pressure surfaces in addition to changes in the thickness fields. To adjust both heights and temperatures would therefore require larger energy input than would be indicated by either field alone. This could easily lead to not only improved temperature fields but also to an improvement in the wave amplitudes (see Fig. 1).

#### 7. SUMMARY

The first, and most obvious, conclusion is that the model tends to damp out waves and move them too slowly. It is also possible to determine the size of these effects. In general, they are fairly small, amounting to a few decameters and a few degrees of longitude.

Examination of the mean zonal flow also indicates a probable error in the parameterization of heat with both the heating and cooling terms being set too low. While these seem to be insignificant they represent a bias

that affects virtually every forecast and thus these errors are well worth investigating and eliminating if possible.

In general it can be said that zonal harmonic analyses can be quite revealing. The model's characteristics can be rather precisely defined. In addition, changes to some parameterizations may be directly related to the forecast of certain waves. This wave behavior can then be used in helping to evaluate the usefulness of different models, data sources or initialization techniques.

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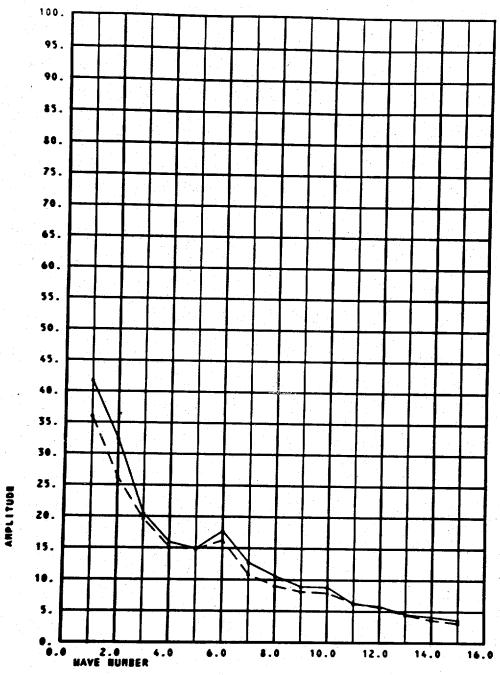


Figure 1A. Amplitude in meters for year 1979 solid line = observed dashed line = 36-hr forecast.

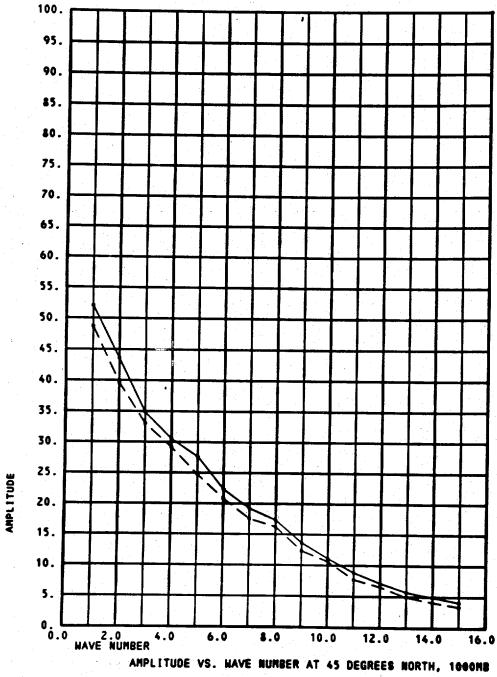
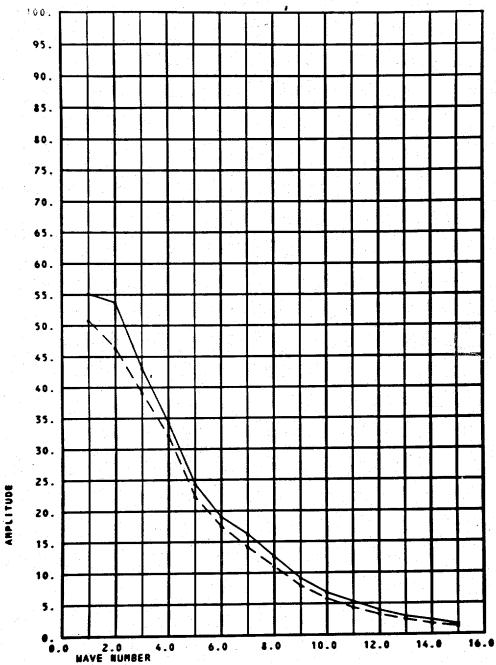


Figure 1B. See Figure 1A.



AMPLITUDE VS. WAVE NUMBER AT 60 DEGREES WORTH. 1000MB Figure 1C. See Figure 1A.

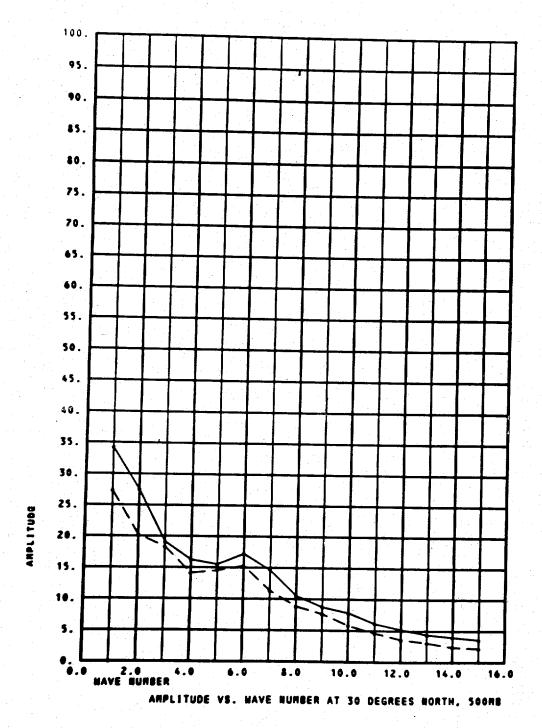


Figure 1D. See Figure 1A.

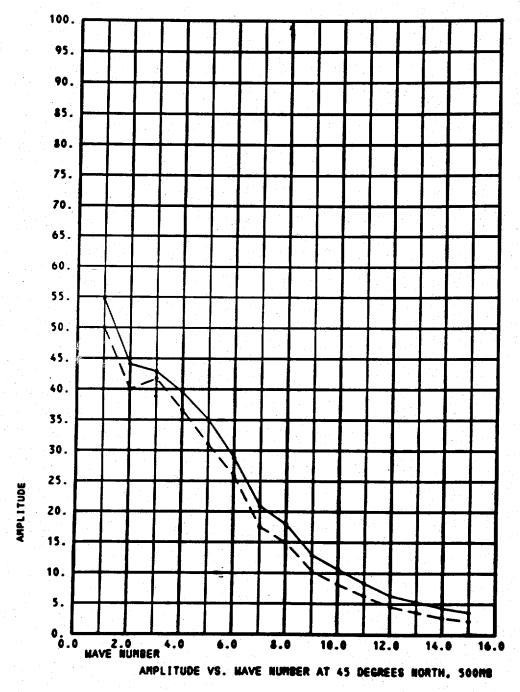


Figure 1E. See Figure 1A.

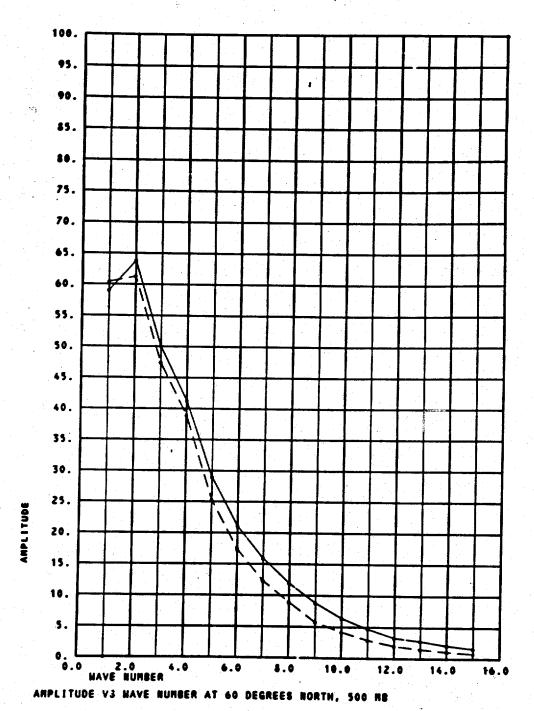


Figure 1F. See Figure 1A.

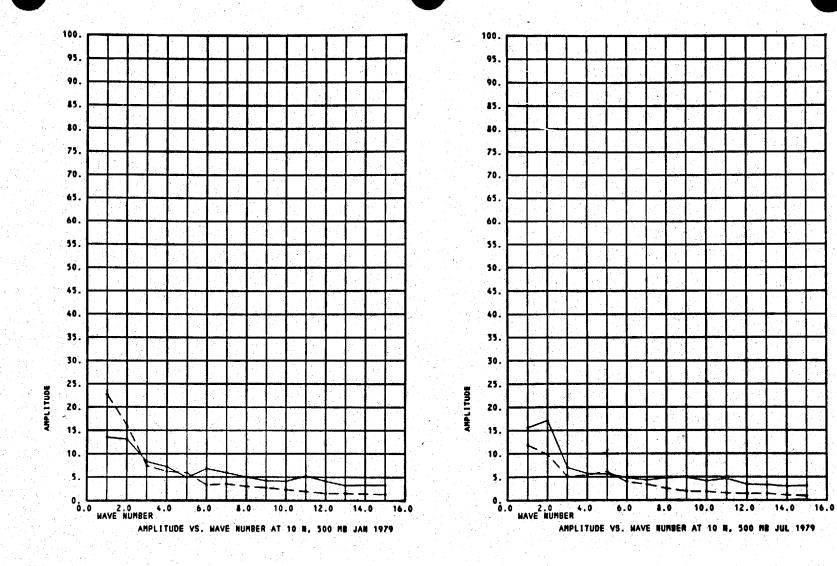


Figure 2A. Amplitude in meters solid line = observed dashed line = 36-hr forecast

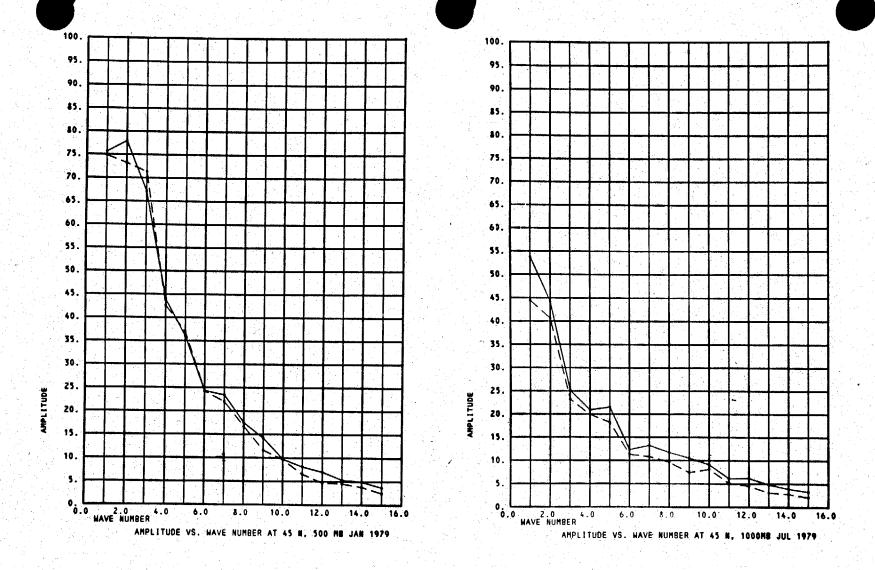


Figure 2B. Amplitude in meters solid line = observed dashed line = 36-hr forecast

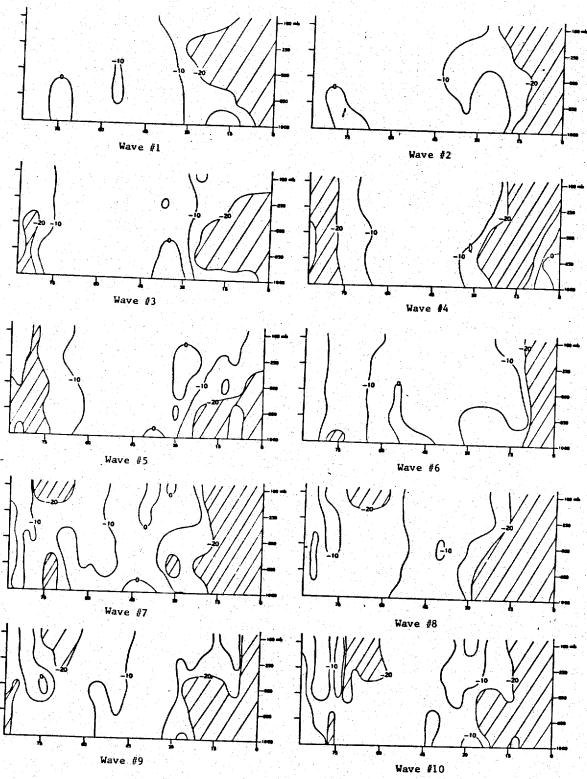


Figure 3A. (See Figure 3B.)

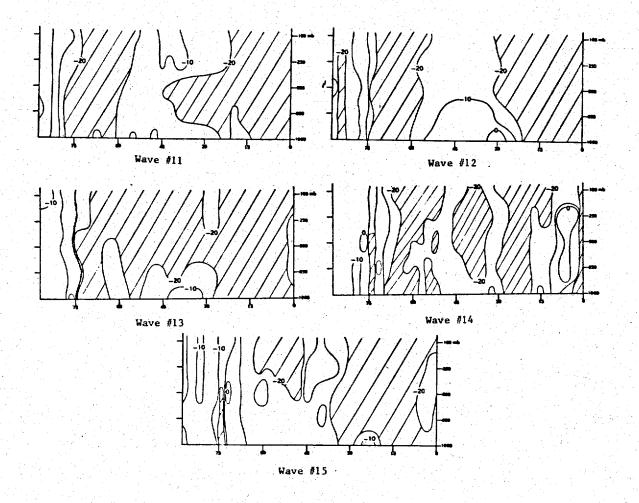


Figure 3B. Phase angle error (F-0), in degrees, by wave number, latitude and level for 36-hr. forecasts during 1979. Shaded area = over  $|20^{\circ}|$ .

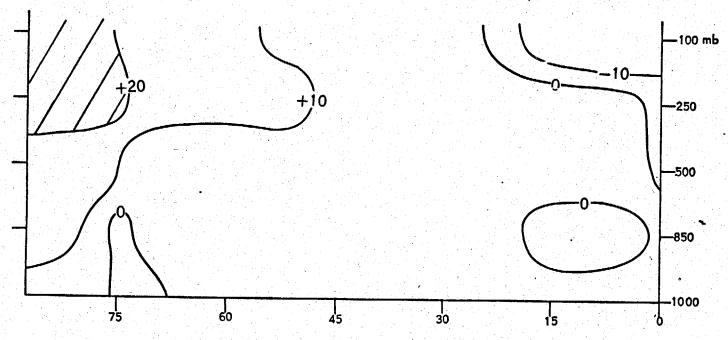


Figure 4. Average error (F-0) in zonal component of 36-hr. forecast of height field, in meters, for May 1979-April 1980.

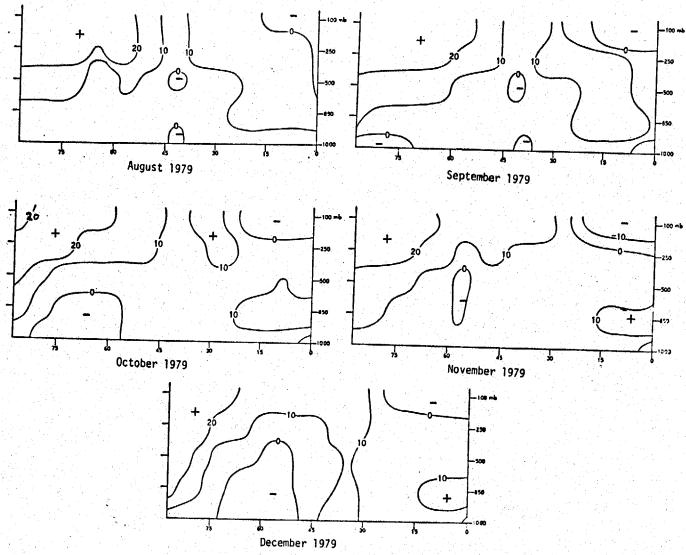


Figure 5A. Error (in meters) of 36-hr. forecast of height field by latitude.

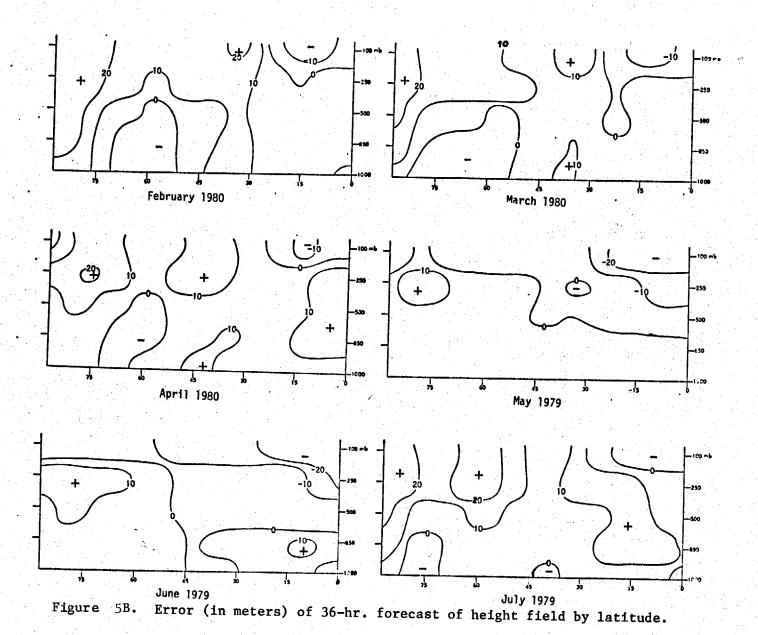


Table 1  $\label{eq:Amplitude} \mbox{Amplitude of Waves in Height Fields at $10^\circ N$, $1979$}$ 

Wave	Number	1000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
		1 - 1	10.5	11 6	11.4	17.0	18.3	15.2	13.5	14.0	18.7
	1	15.1	12.5	11.5 12.4	13.1	16.9	10.9	9.6	8.5	8.7	12 5
	2	11.8	12.3	7.2	7,6	11.3	9.5	7.3	6.0	6.6	10.7
	3	8.9	8.0 5.9	6.3	7.8	9.9	6.5	5.3	5.3	6.4	8.5
	4	8.0 5.0	5.1	5.4	6.3	9.1	5.2	5.1	5.0	5.7	8.5
	5	5.1	5.6	6.4	7.4	10.2	3.9	3.6	3.4	4.1	6.9
	7	3.6	4.4	5.2	6.2	8.7	3.1	3.0	3.1	3.2	5.5
	8	3.6	3.9	4.3	5.1	7.4	3.0	2.6	2.7	2.6	4.2
	9	3.7	3.7	4.1	5.3	7.4	2.4	2.0	2.1	2.1	3.6
	10	3.3	3.3	4.0	4.9	6.6	2.3	2.0	1.9	1.9	3.1
	11	3.7	4.0	4.5	5.4	7.1	2.1	1.6	1.7	1.6	2.7
	12	3.3	3.2	3.7	4.6	6.1	1.9	1.5	1.5	1.4	2.3
	13	2.7	2.8	3.2	4.0	5.3	1.6	1.3	1.4	1.3	2.1
	14	2.4	2.7	3.1	3.7	4.9	1.5	1.2	1.2	1.2	1.9
	15	2.3	2.7	3.1	3.9	5.0	1.4	1.0	1.0	1.0	1.8

Table 2  $\label{eq:amplitude} \mbox{Amplitude of Waves in Height Fields at } 30\mbox{°N}, \ 1979$ 

0001-									
.000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
41.7	37.9	34.2	32.9	48.7	36.0	30.1	27.3	26.6	45.7
32.4	30.2	27.5	26.8						32.8
20.5	16.4	19.1	25.7						31.1
15.9	15.2	16-2	21.9	32.6					29.0
14.9	13.7	15.5	20.2	30.9					26.8
17.7	15.0	17.2	24.6	35.7					31.8
12.7	12.1	14.6	20.4	29.9					26.9
10.6	9.6	10.5	14.4	22.3					19.1
8.9	7.6	8.7	12.5	18.9					16.3
8.8	7.3	7.8	10.5	15.3					12.5
6.2	5.5	6.1	8.3	12.0					9.6
5.8	4.8	5.2	7.1	10.7					7.5
4.5	4.0	4.4	6.0	8.8					5.9
4.1	3.6	4.0	5.3	7.7					5.2
3.6	3.2	3.6	4.8	6.5	3.1	2.5	2.2		4.1
	32.4 20.5 15.9 14.9 17.7 12.7 10.6 8.9 8.8 6.2 5.8 4.5 4.1	41.7 37.9 32.4 30.2 20.5 16.4 15.9 15.2 14.9 13.7 17.7 15.0 12.7 12.1 10.6 9.6 8.9 7.6 8.8 7.3 6.2 5.5 5.8 4.8 4.5 4.0 4.1 3.6	41.7       37.9       34.2         32.4       30.2       27.5         20.5       16.4       19.1         15.9       15.2       16.2         14.9       13.7       15.5         17.7       15.0       17.2         12.7       12.1       14.6         10.6       9.6       10.5         8.9       7.6       8.7         8.8       7.3       7.8         6.2       5.5       6.1         5.8       4.8       5.2         4.5       4.0       4.4         4.1       3.6       4.0	41.7       37.9       34.2       32.9         32.4       30.2       27.5       26.8         20.5       16.4       19.1       25.7         15.9       15.2       16.2       21.9         14.9       13.7       15.5       20.2         17.7       15.0       17.2       24.6         12.7       12.1       14.6       20.4         10.6       9.6       10.5       14.4         8.9       7.6       8.7       12.5         8.8       7.3       7.8       10.5         6.2       5.5       6.1       8.3         5.8       4.8       5.2       7.1         4.5       4.0       4.4       6.0         4.1       3.6       4.0       5.3	41.7       37.9       34.2       32.9       48.7         32.4       30.2       27.5       26.8       38.5         20.5       16.4       19.1       25.7       38.1         15.9       15.2       16.2       21.9       32.6         14.9       13.7       15.5       20.2       30.9         17.7       15.0       17.2       24.6       35.7         12.7       12.1       14.6       20.4       29.9         10.6       9.6       10.5       14.4       22.3         8.9       7.6       8.7       12.5       18.9         8.8       7.3       7.8       10.5       15.3         6.2       5.5       6.1       8.3       12.0         5.8       4.8       5.2       7.1       10.7         4.5       4.0       4.4       6.0       8.8         4.1       3.6       4.0       5.3       7.7	41.7       37.9       34.2       32.9       48.7       36.0         32.4       30.2       27.5       26.8       38.5       26.2         20.5       16.4       19.1       25.7       38.1       19.6         15.9       15.2       16.2       21.9       32.6       15.1         14.9       13.7       15.5       20.2       30.9       14.8         17.7       15.0       17.2       24.6       35.7       16.2         12.7       12.1       14.6       20.4       29.9       10.8         10.6       9.6       10.5       14.4       22.3       9.0         8.9       7.6       8.7       12.5       18.9       8.1         8.8       7.3       7.8       10.5       15.3       7.9         6.2       5.5       6.1       8.3       12.0       6.3         5.8       4.8       5.2       7.1       10.7       5.6         4.5       4.0       4.4       6.0       8.8       4.4         4.1       3.6       4.0       5.3       7.7       3.6	41.7       37.9       34.2       32.9       48.7       36.0       30.1         32.4       30.2       27.5       26.8       38.5       26.2       21.8         20.5       16.4       19.1       25.7       38.1       19.6       16.2         15.9       15.2       16.2       21.9       32.6       15.1       12.8         14.9       13.7       15.5       20.2       30.9       14.8       13.4         17.7       15.0       17.2       24.6       35.7       16.2       13.3         12.7       12.1       14.6       20.4       29.9       10.8       9.3         10.6       9.6       10.5       14.4       22.3       9.0       8.1         8.9       7.6       8.7       12.5       18.9       8.1       7.3         8.8       7.3       7.8       10.5       15.3       7.9       5.7         6.2       5.5       6.1       8.3       12.0       6.3       5.0         5.8       4.8       5.2       7.1       10.7       5.6       4.1         4.5       4.0       4.4       6.0       8.8       4.4       3.7	41.7       37.9       34.2       32.9       48.7       36.0       30.1       27.3         32.4       30.2       27.5       26.8       38.5       26.2       21.8       20.2         20.5       16.4       19.1       25.7       38.1       19.6       16.2       18.3         15.9       15.2       16.2       21.9       32.6       15.1       12.8       14.1         14.9       13.7       15.5       20.2       30.9       14.8       13.4       14.5         17.7       15.0       17.2       24.6       35.7       16.2       13.3       15.3         12.7       12.1       14.6       20.4       29.9       10.8       9.3       11.3         10.6       9.6       10.5       14.4       22.3       9.0       8.1       8.8         8.9       7.6       8.7       12.5       18.9       8.1       7.3       7.6         8.8       7.3       7.8       10.5       15.3       7.9       5.7       5.8         6.2       5.5       6.1       8.3       12.0       6.3       5.0       4.6         5.8       4.8       5.2       7.1	41.7       37.9       34.2       32.9       48.7       36.0       30.1       27.3       26.6         32.4       30.2       27.5       26.8       38.5       26.2       21.8       20.2       21.2         20.5       16.4       19.1       25.7       38.1       19.6       16.2       18.3       23.6         15.9       15.2       16.2       21.9       32.6       15.1       12.8       14.1       19.4         14.9       13.7       15.5       20.2       30.9       14.8       13.4       14.5       18.8         17.7       15.0       17.2       24.6       35.7       16.2       13.3       15.3       21.6         12.7       12.1       14.6       20.4       29.9       10.8       9.3       11.3       27.0         10.6       9.6       10.5       14.4       22.3       9.0       8.1       8.8       12.3         8.9       7.6       8.7       12.5       18.9       8.1       7.3       7.6       10.4         8.8       7.3       7.8       10.5       15.3       7.9       5.7       5.8       8.0         6.2       5.5       6.1 </td

Table 3

Amplitude of Waves in Height Fields at 45°N, 1979

Wave Number	1000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
1	52.0	51.2	54.6	69.3	89.7	48.7	45.7	50.0	64.8	83.5
2	43.5	40.1	44.1	56 - 4	72.9	39.6	35.4	40.0	50.9	67.4
3	34.8	36.2	42.9	56.7	75.0	33.0	35.7	41.7	54.9	74.5
4	30.5	31.9	39.5	56.7	78.5	29.1	29.7	36.6	52.3	72.4
5	27.6	28.4	35.0	51.5	73.0	24.7	25.0	31.0	45.9	66.7
6	22.3	22.7	28.8	43.6	63.3	20.8	20.5	25.8	40.4	60.4
7	19.2	18.3	20.9	31.1	46.0	17.5	15.3	17.5	27.5	42.0
8	17.3	16.0	17.9	26.2	37.3	16.2	14.3	14.9	22.5	33.4
9	13.6	12.0	12.9	18.9	27.7	12.3	10.5	10.3	15.6	24.0
10	11.1	7.3	7.8	10.5	15.3	10.6	8.5	8.1	12.2	18.8
11	8.8	5.5	6.1	8.3	12.0	7.7	6.5	6.2	9.4	14.4
12	7.1	4.8	5.2	7.1	10.7	6.5	5.1	4.5	6.8	10.7
13	5.7	5.1	5.3	7.5	10.0	4.9	4.0	3.6	5.1	7.9
14	4.9	3.6	4.0	5.3	7.7	4.1	3.1	2.7	3.9	6.4
15	4.1	3.2	3.6	4.8	6.5	3.4	2.6	2.2	3.1	5.0

Table 4  $\label{eq:Amplitude} \mbox{Amplitude of Waves in Height Fields at } 60\ensuremath{\,^\circ N}, \ 1979$ 

Wave Number	1000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
1	55.1	53.0	59.0	78.2	107.9	50.8	52.3	60.4	79.7	108.6
2	53.7	52.2	63.8	90.6	120.8	46.6	48.1	61.3	86.1	116.0
3 ,	43.1	43.1	50.0	68.2	92.0	39.1	40.6	47.4	64.6	87.6
<del>4</del> 5	34.2	35.4	41.3	57.7	80.3	32.1	33.5	38.9	55.0	76.8
6	24.3 19.0	24.8	29.0	42.1	59.5	22.3	21.9	25.3	37.5	54.5
7	16.2	18.7 14.7	21.1 15.9	30.1	40.7	17.5	16.2	17.4	25.4	36.6
8	12.6	11.2	11.9	23.3 17.3	31.6	14.1	12.0	12.3	18.7	27.6
9	9.1	8.2	8.8	12.1	22.7 15.0	11.0	9.2	8.9	13.2	19.3
10	6.8	6.1	6.4	9.1	10.8	7.9 5.8	6.4 4.7	5.7	8.1	12.0
1.1	5.3	4.5	4.7	6.5	7.3	4.3	3.4	4.2 2.9	6.1	8.9
12	3.9	3.3	3.3	4.8	5.3	3.2	2.5	2.9	4.1 2.9	6.1
13	2.9	2.6	2.8	3.9	4.1	2.4	1.8	1.5	2.2	4.5 3.3
14	2.3	1.9	2.1	2.9	2.8	1.7	1.3	1.1	1.5	2.3
15	1.6	1.5	1.6	2.2	2.0	1.2	0.9	0.8	1.1	1.7

 $\label{eq:Table 5} \mbox{Amplitude of Waves in Height Fields at 75°N, 1979}$ 

Wave Number	1000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
. 1	62.4	60.9	68.9	89.7	117.7	56.7	57.7	65.2	85.3	112.5
2	40.3	43.1	54.4	79.8	104.9	39.5	41.0	50.8	74.6	99.7
3	27.1	26.1	28.8	40.7	52.1	25.7	22.6	23.9	33.1	44.6
4	17.4	15.8	17.2	24.5	29.8	16.8	13.6	14.3	19.4	25.6
5	10.5	9.4	10.4	15.0	17.2	10.3	7.7	7.4	10.1	13.7
6	6.6	5.4	5.6	8.2	8.8	6.1	4.3	3.7	4.9	6.9
7	4.2	3.4	3.7	5.1	5.0	3.6	2.5	2.1	2.7	3.9
8	2.3	1.9	2.1	2.8	2.4	2.3	1.4	1.1	1.3	1.9
9	1.3	1.1	1.1	1.5	1.1	1.3	0.7	0.6	0.7	1.0
10	0.8	0.7	0.7	0.9	0.7	0.9	0.5	0.4	0.5	0.6
11	0.5	0.5	0.5	0.6	0.6	0.5	0.3	0.3	0.4	0.5
12	0.4	0.4	0.4	0.5	0.6	0.4	0.3	0.3	0.4	0.5
13	0.4	0.4	0.4	0.5	0.6	0.4	0.3	0.3	0.3	0.5
14	0.4	0.3	0.4	0.5	0.6	0.4	0.3	0.3	0.4	0.5
15	0.4	0.3	0.4	0.5	0.5	0.4	0.3	0.3	0.4	0.5

Table 6  $\label{eq:component} \mbox{Average Error by Component of 1979 Height Fields for 36-hr Forecast at 10°N }$ 

Error in A-Component Level

Wave Number	1000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
1	-3.5	-1.6	+1.7	+3.8	+ 8.1	+1.6	+0.7	-2.4	-5.4	- 8.0
2	-5.6	-3.5	-4.9	-8.0	-10.2	-4,4	-7.6	-7.7	-8.3	-12.8
3	-1.3	-1.8	-1.6	-0.9	+ 1.1	-1.3	-2.5	-2.6	-1.1	- 2.2
4	+3.1	+1.4	+1.6	+0.5	- 1.5	-0.0	+0.0	-0.2	+0.7	+ 0.7
5	+3.4	+2.3	+2.5	+3.2	+ 2.0	-1.4	-1.6	-1.4	-1.0	- 0.9
. 6	-0.4	+0.3	+0.7	+1.1	+ 0.5	-3.5	-4.8	-4.8	<b>-</b> 5.7	- 8.1
7	-0.4	-0.8	-1.1	-1.5	- 2.7	-0.8	-1.5	-1.8	-3.2	- 4.8
8	-1.5	-1.1	-1.2	-0.9	- 0.3	-0.2	+0.4	+0.8	+1.2	+ 1.5
9	+0.8	+1.0	+1.2	+2.2	+ 3.4	+0.4	+0.3	-0.0	-0.2	+ 0.1
10	-0.6	-0.9	-1.0	-0.8	- 0.4	-0.1	-0.1	-1.1	-1.9	- 2-4
11	-0.8	-1.3	-1.5	-1.9	- 2.2	+0.9	+1.5	+1.7	+2.1	+ 3.1
12	-1.6	-1.6	-1.7	-1.7	- 1.1	-0.6	+0.1	+0.7	+1.4	+ 2.7
13	-0.3	+0.0	-0.1	+0.0	+ 0.6	-0.3	-1.0	-1.0	-1.5	- 1.9
14	+0.9	+0.9	+0.7	+0.2	- 0.1	+0.5	-0.1	-0.3	-0.4	- 0.9
15	-1.1	-1.2	-1.1	-1.1	- 1.2	+0.7	+0.6	+0.9	+1.4	+ 1.9

Table 7  $\label{eq:table 7} \mbox{Average Error by Component of 1979 Height Fields for 36-hr Forecast at 30 <math display="inline">^{\circ}\mbox{N}$ 

Error in A-Component Level

Wave Number	1000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
•	2 (	10.6	10.7	.1.1 7	+ 5.0	-12.3	-12.5	-12.2	-11.5	-11.5
. 1	-2.6	+0.6	+0.7	+1.7 -3.9	+ 1.1	-12.3 -5.8	-8.3	-6.7	-6.1	-10.6
2	-5.5	-5.4	-5.5 -2.9	-3.9 -5.8	- 7.6	+0.5	-0.5 -1.9	+4.1	+5.2	+ 8.1
	+2.5 +0.9	-0.9 +0.7	+1.3	+0.3	- 1.3	+0.1	+2.2	+0.6	-0.3	+ 0.9
5	+5.5	+7.6	+7.9	+8.5	+ 7.4	-3.7	-3.2	-4.3	-4.1	- 3.8
6	+1.8	+2.9	+1.8	+1.6	+ 3.0	-5.4	-5.6	-6.0	-7.2	- 6.8
7	-2 <b>.</b> 5	-3.7	-3.5	-3.2	- 1.4	+5.8	+4.4	+3.0	+0.8	- 0.4
8	-0.3	-0.6	+0.1	+0.1	- 1.0	+2.0	+2.3	+1.7	+2.0	+ 3.2
9	+2.6	+3.1	+1.9	+1.9	+ 3.1	-2.2	-2.2	-2.5	-1.5	- 0.3
10	-0.6	+0.1	-0.6	-0.4	+ 1.1	+1.0	-0.0	-1.0	-2.1	- 2.9
11	-1.4	-1.8	-1.3	-0.7	- 0.8	+0.9	+0.6	+0.5	+0.0	- 0.7
12	+1.4	+0.3	-0.1	-0.6	- 1.6	+0.9	+1.1	+1.3	+0.7	- 0.1
13	+1.3	+0.8	-0.5	-0.9	- 1.2	+0.7	+0.9	+0.5	-0.3	- 0.1
14	-0.4	+0.2	+0.4	-0.3	- 1.0	-0.7	-0.4	-0.4	-0.5	- 0.4
15	+0.3	-0.3	-0.0	-0.0	+ 0.1	-1.7	-1.1	-1.0	-0.6	- 0.2

Error in A-Component Level

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Wave Number	50 mb 500 mb 250 mb 100 mb 1000 mb	850 mb 500 mb 250 mb 100 mb
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1 2 3 4 5 6 7 8 9 10 11 12 13	6.6	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

Table 9  $\label{eq:average error} \mbox{Average Error by Component of 1979 Height Fields for 36-hr Forecast at 60°N }$ 

Error in A-Component Level

Wave Number	1000 mb	850 mb	500 mb	250 mb	100 mb	1000 mb	850 mb	500 mb	250 mb	100 mb
							- 030 IND		230 mb	100 mb
1	+0.5	+1.6	+2.6	+2.0	+ 0.4	 + 8.1	+ 7.8	+ 6.7	+ 2.2	- 3.4
2	+9.9	+9.7	+9.0	+6.2	+ 6.5	+ 3.8	+ 4.4			- 9.0
3	-2.1	+0.2	-0.9	-2.0	- 0.6	+ 3.1	+ 2 1			2.5
4	-3.7	-2.7	-1.5	-0.7	+ 1.1	+ 3.7	+ 2.4			- 3.7
5	+0.8	+0.4	+0.1	-1.1	- 1.3	- 0.6	+ 1.1			- 3.7
6	+1.3	+1.6	+1.1	+0.8	+ 0.7	+ 1.4	+ 1.8			- 3.7
7	-0.2	-1.3	-1.5	-1.7	- 2.2	- 3.0	- 3.7			1.9
8	+0.6	-1.0	-1.3	-1.4	- 1.3	- 0.1	- 0.0			0.4
9	-0.6	+0.6	+1.0	+0.9	+ 1.0	+ 0.1	+ 0.3			0.6
10	-0.3	+0.3	+0.9	+1.2	+ 1.3	+ 0.6	- 0.2			0.1
11	-0.7	-0.8	-0.7	-0.5	- 0.3	- 0.9	- 0.4			- 0.9
12	+0.1	-0.1	-0.1	-0.0	- 0.0	- 0.3	- 0.2			- 0.2
13	+0.9	-0.3	-0.9	-1.1	- 1.4	- 0:4	- 0.7			0.1
14	-0.0	+0.1	+0.1	-0.0	- 0.1	+ 0.1	+ 0.2			0.7
15	+0.3	+0.4	+0.3	+0.2	- 0.0	+ 0.3	- 0.0			0.4

Table 10

Average Error in Zonal Mean for 36-hr Height Fields in 1979

Latitude	1000 mb	850 mb	500 mb	250 mb	100 mb
O°N	-7.0	+ 7.4	0.0		
2.5°	-0.7	+12.2	- 0.8	- 1.2	-13.4
5.0°	+1.3	+13.8	+ 3.4	+ 2.1	-12.6
7.5°	+3.7	+13.0	+ 4.9	+ 3.3	-11.2
10.0°	+5.7	+14.6	+ 6.9	+ 4.3	-14.2
12.5°	+5.6		+ 7.3	+ 4.4	-16.0
15.0°	+6.4	+12.1 +12.0	+ 5.8	+ 4.4	-13.8
17.5°	+6.3		+ 5.4	+ 3.8	-14.9
20.0°	+6.0	+11.0	+ 5.0	+ 4.2	-10.0
22.5°	+4.9	+ 9.4	+ 4.3	+ 3.4	- 8.2
25.0°	+3.6	+ 8.2	+ 3.3	+ 4.8	- 5.4
27.5°	+3.6	+ 7.1	+ 3.2	+ 6.5	+ 0.1
30.0°	+4.2	+ 6.9	+ 3.9	+ 8.5	- 5.0
32.5°	+5.4	+ 7.6	+ 4.7	+ 9.7	+ 8.2
35.0°	+6.0	+ 8.4	+ 5.2	+ 9.6	+ 9.3
37.5°	+5.9	+ 8.2	+ 4.8	+ 9.0	+ 9.1
40.0°	+5.6	+ 6.7	+ 2.7	+ 7.7	+ 8.0
42.5°		+ 5.0	+ 1.8	+ 6.3	+ 6.6
45.0	+5.5	+ 4.4	+ 1.5	+ 6.1	+ 6.1
47.5°	+5.3	+ 4.7	+ 2.5	+ 7.6	+ 6.9
50.0°	+4.8	+ 4.4	+ 3.9	+ 9.7	+ 8.2
52.5°	+4.6	+ 4.7	+ 4.3	+11.1	+10.1
55.0°	+3.5	+ 3.3	+ 3.9	+11.3	+ 9.6
57.5°	+2.1	+ 0.9	+ 2.1	+10.6	+10.0
60.0°	+1.8	+ 0.0	+ 1.8	+11.1	+11.3
62.5°	+3.3	+ 0.5	+ 2.7	+12.5	+12.8
	+2.2	+ 1.5	+ 4.1	+13.9	+13.9
65.0°	+1.5	+ 1.9	+ 5.0	+14.7	+14.6
67.5°	+0.3	+ 1.3	+ 5.6	+15.4	+15.3
70.0°	-0.6	+ 0.8	+ 6.5	+16.6	+16.6
72.5°	-0.6	+ 1.1	+ 8.0	+18.3	+18.2
75.0°	-0.3	- 2.2	+ 9.6	+20.2	+17.5
77.5°	+0.2	+ 4.2	+11.4	+22.1	+22.1
80.0°	8.0+	+ 6.7	+13.3	+24.0	+23.7
82.5°	+1.8	+ 9.3	+15.3	+26.0	+25.0
85.0°	+2.8	+11 2	+17.0	+27.9	+25.7
87.5°	+3.7	+12.4	+18.3	+29.4	+26.2